MATHEMATICAL THINKING ABILITY OF 7th GRADE STUDENTS IN SOCIAL ARITHMETIC LEARNING THROUGH DISCOVERY LEARNING MODEL

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Abstract

The ability of students is obtained through the learning process of school subject matter in their classes. In particular, learning mathematics has an important role in facilitating students in obtaining various abilities, one of which is ability to think mathematically, an ability that has an important role for students in their lives, both in class and in their daily lives. This study aims to describe the mathematical thinking ability of 7th grade junior high school students in mathematics through a discovery learning model based on Stacey's classification, namely specialization, conjecture, generalization, and convincing. The research method is through qualitative studies by analyzing the answers of students who have gone through social arithmetic learning using discovery learning models. The results of the analysis obtained the findings of students' mathematical thinking ability in terms of specialization, conjecture, generalization, and convincing abilities, students were only identified using the conjecture, generalization, convincing abilities in solving problems/questions given by the educator, it does not appear that specialization ability are seen in all students, then there are problems in taking conjectures, where students are wrong in giving conjectures, this is due to the absence of errors in processing word/sentence information in the questions (word problem). The implication of this problem is that students must be trained in processing the information contained in story problems that involve interpreting words into mathematical symbols.

Keywords: Discovery Learning, Mathematical thinking ability, social arithmetic, Word Problem.

1. Introduction

The ability of students is obtained through the learning process of school subject matter in their classes. In particular, learning mathematics has an important role in facilitating students in acquiring various abilities, one of which is the ability to think mathematically, an ability that has an important role for students in their lives, both in class and in their daily lives. This is supported by the opinion of experts on the notion of mathematical thinking, including the opinion of Mason (Nepal, 2016: 46) which provides an understanding of mathematical thinking as a dynamic process that allows individuals to increase the complexity of ideas they can have and expand understanding. Then, the opinion of Schoenfeld (1992: 31) which states that thinking about mathematics is a broad ability, not only a set of skills in mathematics, but also how a person views their job, and furthermore, how one views the world. Kulm (Argyle, 2012: 23) states that the phrase thinking about mathematics and think when doing mathematics is used to describe activities that are considered mathematical thinking, so that the writer equates the phrase thinking about mathematics which is expressed by Schoenfeld with the phrase mathematical thinking.

Another opinion from Henderson, et. al. (2002: 186) defines mathematical thinking as "Applying mathematical techniques, concepts, and processes, either explicitly or implicitly, in problem solving". It seems that Henderson's understanding is narrower than the previous understanding which gives the meaning of mathematical thinking in a broad scope. Next, the notion of mathematical thinking from Stacey (2005) has a more specific scope, namely in the scope of classroom learning, Stacey stated that mathematical thinking is an ability needed by a teacher in managing her class. Meanwhile, Mason, et al. (2010) states that in mathematical thinking there are four abilities, namely specialization, conjecture, generalization, and convincing. In line with Mason, Stacey (2006) divides the stages of mathematical thinking ability to solve problems into four fundamental processes in two paired stages, namely: specialization and generalization, and guessing and convincing. Meanwhile, according to Tall (2002: 20) the ability to think mathematically includes components such as abstraction, synthesis, generalization, modeling, problem solving, and evidence. The ability to think mathematically is a way for students to acquire mathematical knowledge facilitated by a learning model applied by a teacher in a class.

In a mathematics learning class, direct obstruction of a student's mathematical thinking ability will also hinder the problem solving given by the teacher. This was revealed in a study by Salado, et. al. (2018) which states that one of the mathematical thinking ability affects the ability of students to translate questions into numbers and / or mathematical symbols in the process of solving mathematical problems.

The process of problem solving by students should be directed to the correct procedure to obtain the desired results. Furthermore, Mustafa, et al (2019) stated that the problem solving process of students can be facilitated properly if in learning to implement learning that facilitates students to be actively involved in the learning process. The learning model that can facilitate students to be actively involved in the learning process includes the discovery learning model, as mentioned by Schifter and Fosnot (Fatade et al, 2013: 29) that learning mathematics with discovery learning models focuses on problem solving and conceptual understanding rather than computational exercise. It also increases learners' confidence in their own math skills.

In contrast to the opinion of Schifter and Fosnot, Kirschner, et al. (2006) stated that the discovery learning model as one of the models with minimum guidance has failed in constructing learners' knowledge in a classroom learning. On the other hand, in research related to the implementation of discovery learning models, Saefuloh, et al (2020: 31) stated that the application of the learning model with minimum guidance in mathematics learning experienced several problems. These problems include students with moderate mathematical initial abilities and low difficulty in following learning with this model. Then, there is no significant difference in increasing the problem-solving ability of students who learn through the learning model through minimum guidance compared to students who learn through conventional models. From this statement, there is a conflict between theory and reality, where in theory the discovery learning model can facilitate the acquisition of learners' learning objectives but in this study it is not fulfilled.

The description of mathematical thinking ability can be traced through the problem solving process carried out by students in their class. Mustafa, et al (2019) who examined how the description of students 'mathematical thinking abilities in mathematics learning stated that to understand students' mathematical thinking ability, it could be seen from the stages students took in solving math problems. According to Mustafa, et al. (2019: 118) consists of: problem identification, problem categorization, drawing conclusions from the problem.

Based on the description above, the exploration of mathematical thinking ability wrapped in the application of the discovery learning model in social arithmetic learning becomes an important theme to study. Then, Stacey's unique framework regarding the ability to think mathematically within the scope of the classroom is considered in determining the reference for mathematical thinking ability in this study. How to describe students' mathematical thinking ability (referring to Stacey's framework) in mathematics through discovery learning models is investigated in this study.

Based on the background above, the purpose of this study is to describe the mathematical thinking ability of students in mathematics learning through a discovery learning model that is in line with the constructivism learning paradigm. Then, the research question of this research is how to describe the mathematical thinking ability of 7th graders in mathematics learning in terms of Stacey's framework (specialization, generalization, conjecture, and convincing)? From the results of the description through comparison of fact findings in the field with the reference theory, it can be a reinforcement or a correction to the previous theory so that it can become a new theory in the scientific hierarchy.

2. Literature Review

2.1. Ability to Think Mathematically

Audi (2004: 86), provides understanding of thinking as a process that involves a series of events and is based on a mental event, such as considering a proposition. Based on the understanding of thinking from Audi, it can be understood that thinking is broader than just perception, thinking involves a consideration of a proposition that is coherent with epistemological norms so that knowledge is produced (Wolenski, 2004: 36).

In the field of mathematics, the term "Thinking" uses a more specific vocabulary, namely "Thinking Mathematically", in mathematical thinking the mental events in question can be in the form of a specialization process, conveying conjectures, generalization, and providing proof of a problem which is a component in

mathematical thinking. Some experts give the understanding of mathematical thinking in a broader sense, such as the notion of Keith (Uyangör, 2019: 2) which emphasizes that mathematical thinking is a process that helps us to better understand information about the world in which we live and maximize our choices.

Then, Mason, et al. (Nepal, 2016: 46) provides an understanding of mathematical thinking, which is a dynamic process that allows us to increase the complexity of ideas that we can have and expand our understanding. There is another, more specific understanding of mathematical thinking conveyed by Henderson (2002: 186) that mathematical thinking is an explicit or implicit application of mathematical techniques, concepts, and processes in problem solving.

Scusa (2008: 22), adapting from NCTM, states five characteristics of Mathematical Thinking Ability, namely: (1) Connection, a student who succeeds in making mathematical connections, (2) Representation, a student who is successful in representation, (3)) Communication, a student who successfully communicates mathematically, (4) Reasoning and Evidence, a student who is successful in reasoning and proving, (5) Problem Solving, a student who is a successful problem solver.

Meanwhile, Stacey (2006) divides the ability to think mathematically to solve problems into four fundamental processes in two paired stages, and often the Ability to Think Mathematics is carried out from the selection among these processes, namely: Specialization and Generalization, Conjecture and Convince/Prove, The four processes can also be found in Mason's book, Thinking Mathematically.

Furthermore, the Mathematical Thinking Ability according to Mason, et al. (2010) consists of three different phases, namely: entry, attack, and review, in these three stages there is an emotional state: starting, engaging, thinking, continuing, building insight, be skeptical, ponder. Then, of the three phases, what should get more attention is the entry phase because this phase is the basis for carrying out the attack phase, and the review phase because it is this phase that is often less attention in the knowledge construction process, while this phase is the most loaded phase. education.

2.2. Discovery Learning Model

Starting from the belief of Jean Piaget (Chase & Abrahamson, 2017) which states that students are individuals who actively build their knowledge. Furthermore, this understanding has developed into one of the streams in education, namely the flow of constructivism, in relation to the management of learning in classrooms, one of the representations of constructivist learning models is the discovery learning model.

According to Hosnan (Andra, M., H., et al, 2017: 27) the discovery learning model is a learning model to develop learning methods for students actively by discovering themselves, investigating themselves, and following the results that will be obtained by students. more durable in memory, so that this knowledge will not be easily forgotten by students. According to Eggen and Kauchak (Lestari, W., 2017: 68) the weaknesses of the discovery learning model tend to take up more time and if students do not listen as carefully as they should, they often have wrong conceptions about the topics they are studying. Meanwhile, the advantages of the discovery learning model are that if this learning model is implemented properly it will produce a deep understanding of concepts in students and produce good long-term storage, and encourage students to think critically.

Kurniasih & Sani (Andra, M., H., et al, 2017: 27) suggest the operational steps of the discovery learning model, which are as follows: (1) Stimulation. At this stage students are faced with something that causes confusion, then proceed not to give generalizations, in order to arise the desire to investigate themselves. The teacher can start by asking questions, suggested reading books, and other studies that lead to problem solving preparation. (2) Problem statement. The teacher provides the opportunity for students to identify problems that are relevant to the learning material, then one of them is selected and formulated in the form of a hypothesis. (3) Data collection. This stage students are given the opportunity to collect a variety of relevant information, read literature, observe objects, interviews, conduct their own trials to answer questions or prove whether a hypothesis is true. (4) Data processing. Data processing is the activity of processing data and information that has been obtained by students through interviews, observations and so on. This stage serves as the formation of concepts and generalizations, so that students will get new knowledge from alternative answers that need to be proven logically. (5) Verification. At this stage students carry out careful examinations to prove whether or not the predetermined hypothesis is true or not with alternative findings and associated with the results of data processing. (6) Generalization. The stage of generalization/drawing conclusions is the process of drawing a conclusion that can be used as a general principle and applies to all the same events or problems, taking into account the results of the verification.

3. Research Methods

This study aims to understand how the 7th grade students' mathematical thinking ability in mathematics through discovery learning models. Based on this objective, this research was conducted using a qualitative research approach, with a case study design to obtain a specific description of the object under study which in the process requires an in-depth study of real events in the field, as stated by Fraenkel, JR, Wallen, N., Hyun, H (2012: 426) "Research studies that investigate the quality of relationships, activities, situations, or material are often referred to as qualitative research". Then, Teppo, A., R (1998) states "Qualitative research focuses on the process, meaning, and nature of reality that is socially shaped and provides insight into the phenomenon being studied that cannot be obtained by other means".

A total of eight grade 7 students were selected as a sample consisting of three men and four women who came from one of the junior high schools (SMP) in Bandung and were selected purposively. Data collection was carried out by giving formative questions of 5 social arithmetic questions that must be solved in a lesson. The work of students in answering the problems presented in the study was analyzed by coding and constant comparison techniques (Lacey, A., Luff, D., 2001) by school researchers and expert teachers. Several cases representing the results of the analysis were validated by conducting interviews in the form of confirmation to students.

4. Result and Discussion

There are four types of mathematical thinking skills based on Stacey's framework, namely specialization, conjecture, generalization, and convincing. The following is a description of the thinking skills of students in solving the problems presented.

Table 1. Students' Mathematical Thinking Ability in Solving Problems

Ability to think mathematically	Description
Specialization	Choose clear or systematic examples and examine these examples on a problem to understand and interpret the status of a problem
Conjecture	Making linguistic or mathematical estimates, formulating mathematics, producing the results of conjectures, and determining relevant hypotheses in this process, by examining a sufficient number of examples, finding relationships and patterns, and producing results that depart from the discovery.
Generalization	Estimating about a broader situation by acting on several examples, or can be expressed as a search for patterns / relationships.
Convincing	Finds and communicates reasons why something is true

The results of the analysis obtained through a review of the students' answers by triangulation through interviews with students, which then made a comparison to the theory used as a reference (Stacey's framework) shows that there are two mathematical thinking skills used by students in solving the problems posed. The ability to think is conjecture and generalization. No answers were found that could be categorized on the ability to specialize and be convincing. as for the questions given are as follows:

Questions:

1) There are two traders who usually hang out every day in front of the Melati Shop, namely Mr. Asep as a chicken porridge trader and Mr. Atang as a meatball seller. Every morning Pak Asep goes to the market to shop for staples and spends Rp. 800,000. With these raw materials, Mr. Asep is able to make 120 portions of chicken porridge and sells it for Rp. 8,000 per portion. On that day, it was raining at the place where Pak Asep was selling, so that only 90 portions of porridge were sold. Meanwhile, Mr. Atang spends Rp. 1,000,000 every day to shop for the raw materials for the meatballs. With these raw materials, Pak Atang is able to make an average of 130 servings at a price of IDR 10,000 per serving. On that day Pak Atang was able to sell 110 portions of meatballs.

Create a mathematical model that states the relationship between income, expense and profit or loss in the illustration above! And express the value of the gain or loss in the illustration!

2) Pak Dendi plans to build a shoe production business. To meet his capital needs, Pak Dendi plans to borrow money from the bank of IDR 200,000,000 with a loan period of 1 year (12 months). There are two banks that offer capital assistance to Pak Dendi.

The terms of the loan interest rate at Bank A are 20% per year.

The terms of the loan interest rate at Bank B are 2% per month.

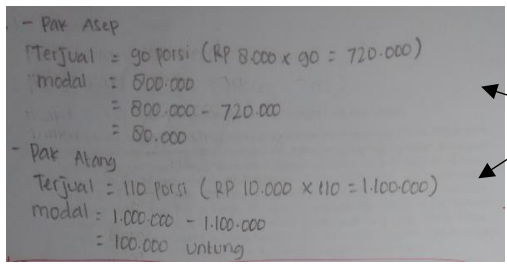
Which bank is more profitable for Mr. Dendi?

The ability to carry out conjectures and generalizations can be seen in all students' answers, in solving problems number 1 and 2, namely determining a mathematical model which states the relationship between income, expenditure and profit or loss on the illustration of the problem and states the value of profit or loss, students can use knowledge of algebraic operations of addition / subtraction

on integers by first understanding the problem in the problem, namely how the formula / equation states the relationship between income, expense, profit / loss in a sale. However, the formal way by using the formulas that have been learned is not used by most students.

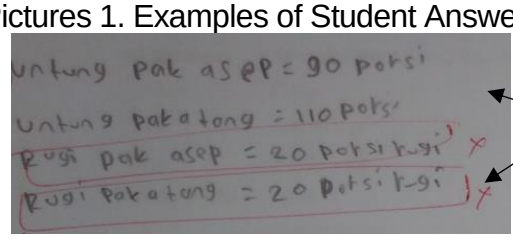
To solve question number 1, the majority of students chose to use the way they think / estimate is correct in solving it, namely by making conjectures (conjectures) based on events that have been experienced before in their daily life, then they (students) apply to the problem in the question (generalization) to answer it. However, from the answers presented, not all students meet the correct solution, although the use of information from everyday life experiences can help the process of achieving solutions. Failure to get the correct solution is because students are wrong in determining the conjecture which then ends in errors in making generalizations.

Here's one example of a student's answer:



conjecture and generalization (true answer)

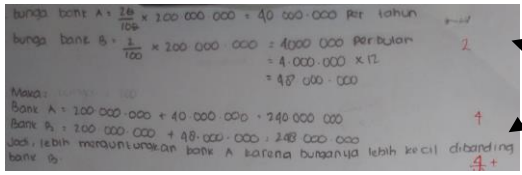
Pictures 1. Examples of Student Answers



conjecture and generalization (false answer)

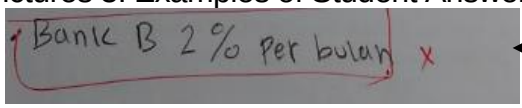
Pictures 2. Examples of Student Answers

From pictures 1 and 2 above, the student answers by making conjectures (conjectures) based on events that have been experienced before in his daily life, then he (students) applies to the problems in these questions (generalizations) to answer them. The student along with 4 other students answered the problem correctly. Other students answer by making the assumption (conjecture) that the profit is if all merchandise is sold out. The deposition of goods (unsold goods) will be a loss for traders. The student also answers by making a supposition (conjecture) that the loss is calculated from the unsold residual merchandise. This happens because students base the wrong information on the questions, students cannot select which information is needed and which is not needed in problem solving. This case is an example of a case of over generalization that has the potential for students' misconceptions about subject matter (Pinter, H., H., et.al, 2017: 11), so that the equations / formulas used do not match the equations / formulas used should be: profit = selling price-buying price or loss = buying price-selling price. (Wintarti, A., et al., 2008: 112).



Conjecture, generalization, and convincing (true answer)

Pictures 3. Examples of Student Answers



Pictures 4. Examples of Student Answers

From figures 3 and 4 above, the student answers by making conjectures (conjectures) based on events that have been experienced before in his daily life, then he (students) applies to the problems in these questions (generalization) to answer them and (convincing) with compare the results. This student along with five other students answered the problem correctly. Other students answer by making assumptions (conjectures) that what is profitable is seen from the interest percentage set, this is the result of problems when solving word problems in social arithmetic learning which is one of the main problems in learning mathematics (Gros, H., et. al., 2020: 16).

In the interview, it was confirmed about this (taking the conjecture), the following was the transcript of the interview with several participants,

Interview transcript 1
Interviewer: "Is the formula for calculating profit / loss memorized?"
Respondent: "forgot sir"
Interviewer: "Then, how do you answer that question?"
Respondent: "Subtracts capital from sales proceeds"
Interview transcript 2
Interviewer: "Is the formula for calculating profit / loss memorized?"
Respondent: "don't know sir"
Interviewer: "Then, how do you answer that question?"
Respondent: "saw what was sold"

From the interview descriptions presented above, it appears that the reason students use their own knowledge (different from learning sources) is due to reasons of forgetting / not knowing the formal formula.

5. Conclusions and suggestions

From the results of the analysis, it can be concluded that students' mathematical thinking skills in terms of thinking skills, students use conjectural thinking skills, generalizations, and convincing in solving problems / questions given by educators, it does not appear that specialization skills are seen in all students, then there are problems in taking conjectures, where students are wrong in giving conjectures, this is due to the absence of errors in processing sentence information in the questions.

The implication of this problem is that students must be trained in processing the information contained in story problems that involve interpreting words into mathematical symbols.

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